Cubic Action in Double Field Theory

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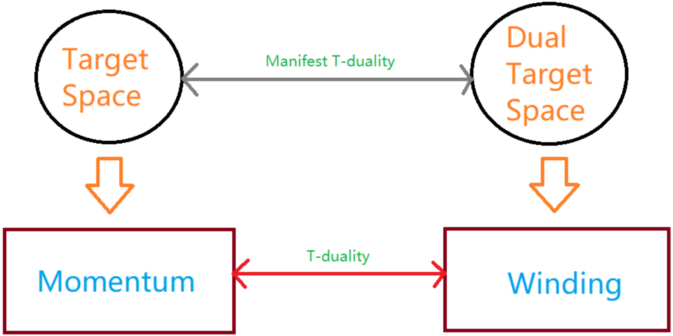
Manifest T-duality



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- the constraint is $\partial_M \partial^M f = -\lambda f$, where $\partial_M \equiv \begin{pmatrix} \tilde{\partial}^m & \partial_m \end{pmatrix}^T$, the doubled indices by $M = 1, 2, \cdots, 2d$, the doubled indices raised or lowered by the O(d, d) metric

$$\eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{1}$$

Quadratic Term

- integration by part shows the constraint $K_A + K_B = 0$ for the quadratic term

$$\int [dxd\tilde{x}]\ A * B. \tag{2}$$

-

$$K_A K_B = -K_A^2 = -\lambda. \tag{3}$$

- the products are commutative and given by that:

$$\begin{aligned} & A * B \\ \equiv \quad & \sum_{K_A, K_B} A_{K_A} B_{K_B} \exp\big(i(K_A + K_B)X\big) \delta_{K_A K_A, \lambda} \delta_{K_B K_B, \lambda} \delta_{K_A K_B, -\lambda}. \end{aligned}$$

Gauge Transformation

- the gauge transformations are:

$$\begin{aligned} &\delta_\xi \mathcal{H}^{MN} \\ =\ & \xi^P * \partial_P \mathcal{H}^{MN} + \left(\partial^M \xi_P - \partial_P \xi^M\right) * \mathcal{H}^{PN} \\ & +\left(\partial^N \xi_P - \partial_P \xi^N\right) * \mathcal{H}^{MP}; \\ &\delta_\xi \Phi \\ =\ & -\frac{1}{2}\partial_M \xi^M + \xi^M * \partial_M \Phi, \end{aligned} \tag{4}$$

where the generalized metric and the scalar dilaton:

$$\begin{aligned} \mathcal{H}_{MN} &\equiv \begin{pmatrix} g^{-1} & -g^{-1} * \mathrm{b} \\ \mathrm{b} * g^{-1} & g - \mathrm{b} * g^{-1} * \mathrm{b} \end{pmatrix}; \\ \Phi &= e^{-2\phi} * \sqrt{|\det g|}. \end{aligned} \tag{5}$$

- the λ does not appear in the gauge transformation explicitly

Field Content

- For $N_L + N_R = 2$, we have three choices $(N_L, N_R) = (1,1); (2,0); (0,2)$.
- The first choice is a massless state with a symmetric traceless tensor field h_{jk}, a scalar field ϕ, and an anti-symmetric tensor field b_{jk}.
- Other choices are the massive states with a symmetric traceless tensor field h_{jk}, a scalar field ϕ, and a one-form gauge field A_j.
- We will make a field correspondence for the massive state as in the following $b_{jk} = -(\tilde{\partial}_j A_k - \tilde{\partial}_k A_j) + (\partial_k A_j - \partial_j A_k)$ with $\delta A_j = \epsilon_j = \tilde{\epsilon}_j$.

Cubic Action

- We introduce the following additional terms to obtain the gauge invariant theory for $(N_L, N_R) = (2,0); (0,2)$:

  (1) Quadratic Term

$$\tilde{S}^{(2)}_{add} = \frac{1}{16\pi G_N}\int [dxd\tilde{x}]\left(-\frac{\lambda}{4}b^{jk}b_{\mu\nu} - \frac{\lambda}{4}h^{jk}h_{jk} + 4\lambda\Phi^2\right); \quad (6)$$

  (2) Cubic Term

$$\begin{aligned}&\tilde{S}^{(3)}_{add}\\ = \quad & \frac{\lambda}{16\pi G_N}\int [dxd\tilde{x}]\left[\frac{1}{4}\Phi e^{lk}e_{lk} - 4\Phi\Phi\Phi - \frac{1}{2}A^m e_{mk}D_l e^{lk}\right.\\ & \left. -2A_l e^{lk}\bar{D}_k\Phi + A^m\bar{D}_k(A_m)D_l(e^{lk}) + 2\lambda A_l A^l\Phi\right], \qquad (7)\end{aligned}$$

  where

$$D_k \equiv \partial_k - \tilde{\partial}_k; \qquad \bar{D}_k \equiv \partial_k + \tilde{\partial}_k. \qquad (8)$$

$d = 1$

- For one double direction, the constraints are $K_A^2 = K_B^2 = \lambda$, which give the momenta:

$$K_A = \begin{pmatrix} a \\ \frac{\lambda}{2a} \end{pmatrix}; \qquad K_B = \begin{pmatrix} b \\ \frac{\lambda}{2b} \end{pmatrix}, \tag{9}$$

  where a and b are real-valued.
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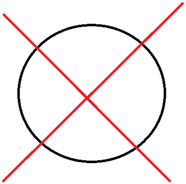
- This result implies



Thank you!